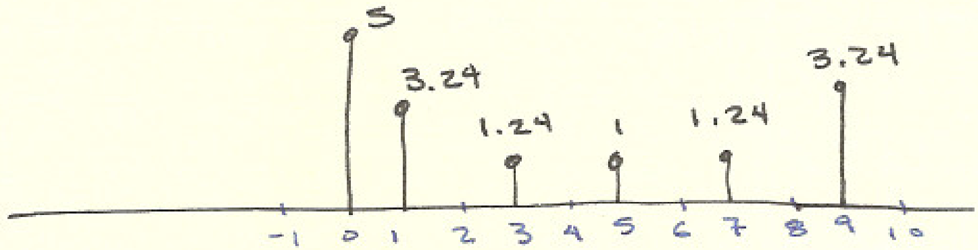
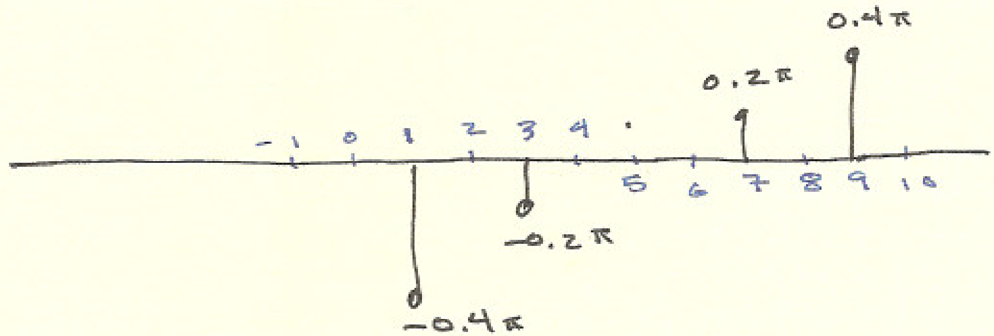
Ex: Same problem as yesterday, but w/ $N=10$ rather than 5.





Properties of DFT.

Linearity

If $x_3[n] = x_1[n] + x_2[n]$

then $X_3[k] = X_1[k] + X_2[k]$

length for $x_3[n]$, $N_3 = \max(N_1, N_2)$

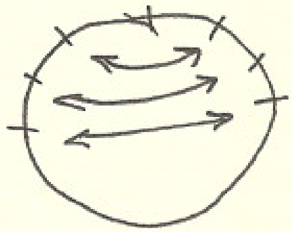
All three must be computed with the same N.

Circular Folding

$$x[-n \bmod N] = \begin{cases} x[0], & n=0 \\ x[N-n], & 1 \le n \le N-1 \end{cases}$$

Then the corrisponding DFT

$$x[-k \bmod N] = \begin{cases} x[0], & k=0 \\ x[N-k], & 1 \le k \le N-1 \end{cases}$$



Circular Shifting (AKA rotation)

In order to get circularly shifted seq first we have to get $x[n]$ and then shift it linearly

$$\tilde{x}[n-m] = x[(n-m) \bmod N]$$

$$\text{or} \quad x[n-m] = \begin{cases} \tilde{x}[n-m], & 0 \le n \le N-1 \\ 0, & \text{else where.} \end{cases}$$

~~In~~ $$\text{DFT}\{x[n-m]\} = e^{-j\left(\frac{2\pi k}{N}\right)m} X[k]$$

Circular Convolution

let
$$x_1[n] \longrightarrow X_1[k]$$
$$x_2[n] \longrightarrow X_2[k]$$
$$x_3[n] \longrightarrow X_3[k]$$

If
$$X_3[k] = X_1[k] \cdot X_2[k]$$

Then
$$x_3[n] = x_1[n] \; N \; x_2[n]$$

ⓝ : N-point circular convolution

$$x_3[n] = \sum_{m=0}^{N-1} x_1[m]\, x_2[(n-m) \bmod N]$$

$$; \quad 0 \leq n \leq N-1$$

Multiplication Property.

$$\text{DFT}\{x_1[n]\, x_2[n]\}$$

$$= \frac{1}{N} X_1[k] \; ⓝ \; X_2[k]$$

Parseval's Relation

Energy of a sequence

$$E_x = \sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

EX:

$$x_1[n] = \{\underset{\uparrow}{1}, 2, 2\}$$

$$x_2[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

$$x_1[n] \; ④ \; x_2[n] = ?$$

Method 1 (Direct computation)

$$= \sum_{m=0}^{3} x_1[m] \cdot x_2[(n-m) \bmod 4]$$

$$x_3[0] = \sum_{m=0}^{3} x_1[m]\, x_2[-m \bmod 4]$$

$$= \sum_{m=0}^{3} \{\underset{\uparrow}{1}, 2, 2, 0\} \{\underset{\uparrow}{1}, 4, 3, 2\}$$

$$= 15$$

$$x_3[1] = \sum_{m=0}^{3} x_1[m]\, x_2[(1-m) \bmod 4]$$

$$= \sum_{m=0}^{3} \{1, 2, 2, 0\} \{2, 1, 4, 3\}$$

$$= 12$$

$$x_3[2] = 9$$

$$x_3[3] = 14$$

$$x_3[n] = \{\underset{\uparrow}{15}, 12, 9, 14\}$$

Method 2. (DFT)

$$x_3[n] = x_1[n] \; ④ \; x_2[n]$$

$$X_3[k] = X_1[k]\, X_2[k]$$

$$X_1[k] = \sum_{n=0}^{3} x_1[n]\, W_N^{kn}$$

$$W_4 = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = -j$$

$$x_1[0] = \sum_{m=0}^{3} x_1[n]\, W_4^{0} = 1+2+2 = 5$$

$$x_1[1] = \sum_{m=0}^{3} x_1[n]\, W_4^{n} = 1\cdot(-j)^0 + 2\cdot(-j)^1 + 2(-j)^2 + 0$$
$$= -2j - 1$$

$$x_2[2] = \sum_{n=0}^{\infty} x_1[n]\, W_4^{2n} = 1\cdot(-j)^0 + 2(-j)^2 + 2(-j)^4$$
$$= 1$$

$$x_2[3] = -1 + 2j.$$

$$x_1[k] = \{\underset{\uparrow}{5}, -2j-1, 1, -1+2j\}$$

$$x_2[k] = \{\underset{\uparrow}{10}, -2+2j, -2, -2-j2\}$$

$$x_3[k] = x_1[k]\, x_2[k] = \{50, 6+2j, -2, 6-j2\}$$

$$x_3[n] = \frac{1}{4}\sum_{k=0}^{3} x_3[k]\, W_4^{-kn}$$
$$= \{\underset{\uparrow}{15}, 12, 9, 14\}$$

Efficient Computation of DFT.

Fast fourier transform (FFT) is an efficient algorithm for computation of DFT.

Radix-2 Decimation in time fft

Let $N = 2^{\nu}$; where ν is an integer

$\therefore$ N is an even interger.

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \quad ; \quad 0 \le k \le N-1$$

$$= \sum_{even} x[n] W_N^{kn} + \sum_{odd} x[n] W_N^{kn}$$

$$= \sum_{r=0}^{N/2-1} x[2r] W_N^{2rk} + \sum_{r=0}^{N/2-1} x[2r+1] W_N^{(2r+1)k}$$

$$= \sum_{r=0}^{N/2-1} x[2r] (W_N^2)^{rk} + \sum_{r=0}^{N/2-1} x[2r+1] (W_N^2)^{rk} W_N^k$$

$$W_N^2 = e^{-j\frac{2\pi}{N}\cdot 2} = e^{-j\frac{2\pi}{N/2}}$$

$$= W_{N/2}$$

$$= G[k] + W_N^k H[k]$$